

Circular Motion

Vocabulary **Period (T):** The time it take for one full rotation or revolution of an object.
Frequency (f): The number of rotations or revolutions per one second (s).

Period and frequency are reciprocals of each other. In other words,

$$T = 1/f \quad \text{and} \quad f = 1/T$$

Since period is a measure of time, its SI unit is the **second**, while the unit for frequency is the reciprocal of this, or 1 / second. Another way or writing 1 / second is the unit **hertz (Hz)**.

When an object spins in a circle, the distance it travels in one revolution is the circumference of the circle, $2 \pi r$. The time it takes for one revolution is the period, T. Therefore,

$$\text{velocity} = 2 \pi (\text{radius}) / \text{period} \quad \text{or} \quad v = 2 \pi r / T$$



Where v is called the **linear** or **tangential velocity** because at any given time, the velocity is tangent to the circle as shown in the diagram. Although the velocity is constant in magnitude, it is always changing direction, and therefore accelerating.

Centripetal Acceleration and Centripetal Force

An object can move around in a circle with a constant speed yet still be accelerating because its direction is constantly changing. This acceleration, which is always directed in toward the center of the circle, is called **centripetal acceleration**. The magnitude of this acceleration is written as:

$$\text{Centripetal acceleration} = (\text{linear speed})^2 / \text{radius} \quad \text{or} \quad a_c = v^2 / r$$

If a mass is being accelerated toward the center of a circle, it must be acted upon by an unbalanced force that gives it this acceleration. This force, called the **centripetal force**, is “center-seeking” and directed inward toward the center of the circle.

$$\text{Centripetal force} = (\text{mass})(\text{centripetal acceleration}) \quad \text{or} \quad F_c = ma_c = mv^2 / r$$

The units for centripetal acceleration and centripetal force are m/s^2 and N, respectively.

Solved Examples

Example 1: Missy's favorite ride at the County Fair is the rotor, which has a radius of 4.0 m. The ride takes 2.0 s to make on full revolution. a) What is Missy's linear speed on the rotor? b) What is Missy's centripetal acceleration on the rotor?

Solution: The ride takes 2.0 s to make on full revolution, so the period is 2.0 s.

a. Given: $r = 4.0\text{ m}$
 $T = 2.0\text{ s}$

Unknown: $v = ?$

Unknown:

Original equation: $v = 2 \pi r / T$

Substitute: $= 2 \pi (4.0\text{ m}) / 2.0\text{ s}$
 $= 13\text{ m/s}$

b. Given: $v = 13\text{ m/s}$
 $r = 4.0\text{ m}$

Unknown: $a_c = ?$

Strategy:

Original equation: $a_c = v^2 / r$

Substitute: $= (13\text{ m/s})^2 / 4.0\text{ m}$
 $= 42\text{ m/s}^2$

Example 2: Captain Chip, the pilot of a 60,500 kg jet plane, is told that he must remain in a holding pattern over the airport until it is his turn to land. If Captain Chip flies his plane in a circle whose radius is 50.0 km once every 30.0 min, what centripetal force must the air exert against the wings to keep the plane moving in a circle?

Solution: First, convert km to m and min to s.

$$50.0\text{ km} = 5.00 \times 10^4\text{ m}$$

$$30.0\text{ min} = 1.80 \times 10^3\text{ s}$$

Before solving for the centripetal force, find the speed of the airplane.

Given: $T = 1.80 \times 10^3\text{ s}$
 $r = 5.00 \times 10^4\text{ m}$

Unknown: $v = ?$

Strategy:

Original equation: $v = 2 \pi r / T$

Substitute: $= 2 \pi (5.00 \times 10^4\text{ m}) / 1.80 \times 10^3\text{ s}$
 $= 175\text{ m/s}$

Use this speed to solve for the centripetal force.

Given: $m = 60,500\text{ kg}$
 $v = 175\text{ m/s}$
 $r = 5.00 \times 10^4\text{ m}$

Unknown: $F_c = ?$

Strategy:

Original equation: $F_c = m v^2 / r$

Substitute: $= (60,500\text{ kg})(175\text{ m/s})^2 / 5.00 \times 10^4\text{ m}$
 $= 3.71 \times 10^4\text{ N}$

Physics

Circular Motion - Problem Set I -

Name:

Date:

Rotational

Reference:

$$T = \frac{1}{f}$$

$$v = \frac{2\pi r}{T}$$

$$a_c = \frac{v^2}{r}$$

$$F_c = ma_c$$

1. Mark puts on his favorite Backstreet Boys compact disc in his CD player. The standard diameter for a disc is 12cm (0.012m) and a mass of 15 grams (0.015kg). The player spins the disc at 10 m/s linear velocity measured at the outer edge.

- a) what is the acceleration of the disc ?
- b) what is the rotational force associated with this acceleration?

Sketch:

Given:

Solve:

Answer

A) _____

B) _____

2. Earth turns on it's axis about once every 24 hours.

The radius of Earth is 6.4×10^6 meters.

- a) IF some catastrophe suddenly brought Earth to a halt with what speed would all the free bodies living at the equator go flying off? (this will never happen! – but IF it did)
- b) Because Earth is solid it must turn with the same frequency everywhere on the surface. Compare the linear speed at the equator (part a) to the linear speed while standing at one meter away from the North or South poles. Greater, less or same and why?

Sketch:

Given:

Solve:

Answer

A) _____

B) _____

Physics

Name:

Circular Motion - Problem Set I -

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3. A popular demonstration is to swing a bucket of water around in a vertical circle fast enough so that the water doesn't spill out when the pail is directly overhead and upside down. If Emily tries this trick and has an arm length of 60 cm (0.6 meters) - what is the minimum linear speed at which she can swing the bucket and still stay dry?

Sketch:

Given:

Solve:

Answer: _____meters/second

4. To test stamina, astronauts are subjected to many rigorous physical test before they fly in space. One test involves spinning around in a device called a centrifuge. This subjects them to accelerations greater than gravity alone (a.k.a $1 g = 10 \text{ m/s}^2$). With what linear speed does an astronaut have to spin in order to experience 3 g's if the radius arm of the centrifuge is 10.0 meters? b. [convert to miles per hour]

Sketch:

Given:

Solve:

Answer: _____

Answer: _____mi/hr

5. Bianca is making a milkshake in her blender. A 0.01 kg gram strawberry is rapidly spun around the inside of the container with a linear speed of 15 m/s. It is held in place by a centripetal force of 50 N. What is the radius of the blender? [convert to centimeters]

Sketch:

Given:

Solve:

Answer: _____cm